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## DESIGN SENSITIVITY ANALYSIS OF ROTORCRAFT AIRFRAME STRUCTURES FOR VIBRATION REDUCTION

T. Sreekanta Murthy PRC Kentron, Inc. Hampton, VA

#### **ABSTRACT**

As a part of an ongoing NASA/industry rotorcraft structural dynamics program, a study was recently initiated at Langley on optimization of rotorcraft structures for vibration reduction. The objective of this study is to develop practical computational procedures for structural optimization of airframes subject to steady-state vibration response One of the key elements of any such computational proceconstraints. dure is design sensitivity analysis. A method for design sensitivity analysis of airframes under vibration response constraints is presented. The mathematical formulation of the method and its implementation as a new solution sequence in MSC/NASTRAN are described. The results of the application of the method to a simple finite element 'stick' model of the AH-1G helicopter airframe are presented and discussed. Selection of design variables that are most likely to bring about changes in the response at specified locations in the airframe is based on consideration of forced response strain energy. Sensitivity coefficients are determined for the selected design variable set. Constraints on the natural frequencies are also included in addition to the constraints on the steady-state response. Sensitivity coefficients for these constraints are determined. Results of the analysis and insights gained in applying the method to the airframe model are discussed. The general nature of future work to be conducted is described.

#### INTRODUCTION

Excessive vibrations have a detrimental influence on the performance, operation and maintenance of helicopters. The primary source of vibration in the airframe arises from the vibratory airloads acting on the main rotor which are transmitted to the airframe at known discrete frequencies. Vibration continues to be a problem in helicopters despite considerable efforts to reduce it. The problem has been attacked by the use of active and passive vibration control devices, by changes to main rotor system and by airframe design. of vibration control devices involves weight penalties. Alterations to the rotor by modifying blade stiffness and mass distribution are being Airframes are designed to satisfy strength, vibration and performance requirements. Dsign for vibrations is based primarily on previous experience. Selection of the best airframe that meets all the requirements, in particular the vibration requirements, is a difficult It would appear that structural optimization tools, properly brought to bear by the design engineer, would go a long way toward achieving the goal of an analysis capability for designing a low vibration helicopter.

The use of structural optimization in helicopter airframe design for vibration reduction is a relatively new research topic and has only recently been addressed. Work related to "optimization" of helicopter airframe structures is contained primarily in references 1-6. However, only references 5 and 6 use a nonlinear programming approach. Sciarra (1) used a strain energy approach to guide modification of a structure; Done (2) and Sobey (3) used the Vincent Circle approach; Hanson (4) did a comparative study of the above two approaches; Done and Rangacharyulu (5) and Miura and Chargin (6) used a formal optimization approach for airframe design.

As a part of an ongoing NASA/industry rotorcraft structural dynamics program, a study was recently initiated at Langley on optimization of rotorcraft structures for vibration reduction. objective of this study is to develop practical computational procedures for optimization of rotorcraft structures subject to steady-state vibratory loads. One of the key elements in the development of a computational procedure for airframe optimization is design sensitivity analysis. A method for design sensitivity analysis of airframes under steady-state response due to rotor-induced dynamic loads is presented. Constraints on airframe dynamic response displacements and natural frequencies are considered. formulation of the method and its implementation as a new solution The results of the application sequence in MSC/NASTRAN are described. of the method to a simple finite element 'stick' model of the AH-1G helicopter airframe are discussed. The paper concludes with a short discussion of the direction future in-house work in this area is to take.

### DEFINITION OF OPTIMIZATION PROBLEM

The airframe structure of a helicopter is subjected to steady-state rotor-induced harmonic loads acting at the top of the rotor mast. The loads, in general, have six components and occur at frequencies which are integer multiples of the product of the number of blades and the rotor rotational speed. It is assumed that both the magnitude and frequency of the rotor loads acting on the airframe are known and that they are constant during design modifications.

The airframe structure is assumed to have nonuniform stiffness and mass distributions which are functions of the geometry of the structural members. The design variables are taken to be the dimensions which characterize the cross-sectional geometry of a member. particular, for a beam member having a solid rectangular cross-section the design variable would be the depth and height. Selection of design variables in a large airframe structure containing thousands of members is a difficult task. An experienced airframe designer can suggest candidate members that can be permitted to undergo design modification and the extent to which they can be modified. Sciarra (Ref.1) and Hanson (Ref.4) have provided some quidelines in the selection of design variables. In particular it has been shown that the design variables that are most likely to bring about changes in the response at specified locations in the airframe are the ones having maximum forced response strain energy. Using this criterion an initial selection of design variables of an airframe can be made. general, any design change will introduce changes in dynamic response, natural frequencies, mode shapes, static strength, weight, and center of gravity location of an airframe and they in turn indirectly change the performance characteristics of a helicopter as a whole. constraints have to be imposed on the allowable response characteristics to restrict design changes within certain bounds. For the work reported in the paper, only constraints on steady-state dynamic response displacements and natural frequencies are considered.

To complete the definition of the optimization problem, an objective function must be defined. This is not an easy task. Should the airframe weight be the objective function or the dynamic response displacement? If the former is selected as the objective function, can the reduced dynamic response be achieved without increasing the stiffness and hence the mass of an airframe? If the latter is the objective function an optimizer may try to drive the response at a point to zero which may not result in reduction of vibration at other points on an airframe. Because this paper is limited to a study of design sensitivity analysis, these additional considerations are not addressed here.

### DESIGN SENSITIVITY ANALYSIS OF AIRFRAME

In this section formulation of design sensitivity analysis of an airframe with constraints on steady-state dynamic response displacements is presented and equations for determining the sensitivity coefficients are given. Also, pertinent equations used in the study, such as equations for airframe response analysis and expressions for strain energy, are presented.

The equation of motion (state equation) for determining the steady-state dynamic response is given in the Figure (1). The equation is written in matrix form in terms of the coefficient matrices K (stiffness), M (mass), C (damping), and F (force). The magnitude and frequency of the force F are assumed to be known. Steady-state response X occurs at the same frequency as the forcing frequency. The unknown response vector X is obtained by solving a set of simultaneous linearalgebraic equations. The equation of motion for the undamped natural frequencies of an airframe is given. Expressions for modal element strain energy and undamped forced response strain energy are also given in the figure.

To determine the sensitivity coefficients for constraints on the steady-state response X, the design variable b is changed by a small amount db. The structural members associated with the design variables will have new cross-sectional properties and new stiffness, mass and damping matrices for the changed design. Thus, for a small change in a design variable b, new K, M, and C are computed and a new response is generated. The response x for the new design must satisfy the equilibrium requirement h(b,x)=0. A linearized version of this requirement is used to derive an expression for the sensitivity coefficients  $\partial x/\partial b$  as outlined in Figure (2). The matrices on the left-hand side (LHS) of the equation for the sensitivity coefficients are already known from the finite element analysis for a particular design. In the right-hand side (RHS) the change in force due to a change in design is assumed to be zero. Only the changes in the stiffness, mass and damping matrices due to an increment in design have to be computed. The matrices thus formed are assembled and solved as a set of simultaneous linear algebraic equation for the unknowns ax/ab. An incremental form of the equations for sensitivity coefficients is also given in the figure. The size of the matrix on the RHS is dependent on the number of design variables and number of forcing frequencies used in the analysis. The sensitivity coefficients  $\frac{\partial x}{\partial b}$ are obtained in a matrix form with rows corresponding to the number of airframe degrees of freedom and columns corresponding to the number of airframe design variables. The number of matrices of ax/ab depends on the number of load cases considered.

### IMPLEMENTATION OF SENSITIVITY ANALYSIS IN MSC/NASTRAN

NASTRAN is used in the helicopter industry for finite element analysis applications, and therefore it was judged appropriate to implement the sensitivity analysis in that program. A new solution sequence to compute the sensitivity coefficients using NASTRAN Direct Matrix Abstraction Program (DMAP) modules was developed. incremental form of the equation for the sensitivity coefficients for constraints on steady-state dynamic response displacements was implemented using the DMAP modules and incorporated into MSC/NASTRAN. The solution for the sensitivity coefficients is obtained in the sequence shown in Figure (3). The corresponding DMAP modules are also The DMAP program uses the data about design variables and shown there. constraints specified on NASTRAN bulk data cards (DVAR, DVSET, and The data for the stiffness and mass matrices of the airframe generated in a previous finite element analysis are retrieved from the data base using module DBFETCH. Damping was not considered in the current implementation. The program generates new cross-sectional properties of structural members for an increment in design and rearranges the intermediate data using module DSTA. Using modules EMG and DSVG1. AK and AM are computed. The RHS of the equations for sensitivity coefficients is assembled using module ADD. The equations are then solved using the FRRD1 module to obtain the sensitivity Several other DMAP coefficients for the dynamic response constraints. modules, such as SSG2, MODACC, SDR1, SDR2, DSMA, DBSTORE and LMATPRT, are used for pre-and post-processing of data used in the solution sequence and also for organizing the stiffness, mass and sensitivity coefficient matrices in a partitioned form.

Numerical results for sensitivity coefficients for constraints on steady-state dynamic response are obtained as follows. First, the airframe dynamic response is obtained from Rigid Format 68. Then, the solution sequence described above is executed.

## APPLICATION TO AH-1G HELICOPTER AIRFRAME

Description of the AH-1G Airframe:

The airframe structure of the AH-1G helicopter descibed in references 4 and 7 was used for the sensitivity analysis application. The airframe structure with its skin panels removed is shown in Figure The fuselage portion of the airframe is built around two main beams which provide the primary vertical bending stiffness in the fuselage structure. The main beams are tied together by the lower horizontal floors, the forward fuel cell cover, and the engine deck to give the fuselage lateral stiffness. The main rotor pylon provides the structural connection between the main rotor and the fuselage. It is attached to the fuselage through five elastomeric mounts and a lift The lift link is the primary vertical load path and is pinned to the center wing carry-through beam. The engine, gun turret and the landing gear are attached to the fuselage. The wings (not shown) are designed mainly for carrying external loads and are attached to the fuselage on either side. The tailboom is bolted to the fuselage with four attachment fittings. The tailboom is of semimonocoque construction having aluminium skins. stringers and longerons. The vertical fin is connected to the tailboom through the tail rotor mast.

Elastic Line Model of the AH-1G Airframe:

A built-up finite element model of the AH-1G airframe structure is available (Ref. 7). However, for the initial studies on sensitivity analysis which are the subject of this paper, an elastic line or 'stick' model of the AH-1G airframe (Ref. 4) was used. The model is shown in Figure (5). The dynamic characteristics of this elastic line model are similar to those of the built-up model of the airframe. fuselage, tailboom, wings and rotor mast structure of the airframe were modelled with beam elements. Scalar spring elements were used in the pylon support structure. The engine and the gun turret mounts were modelled as rigid bar elements. The NASTRAN finite element model of this airframe consists of 42 beam elements, 13 scalar spring elements and 12 rigid elements. There are 56 grid points in the model for a total of 336 degrees of freedom. After applying multi-point and single-point constraints and omitting massless degrees of freedom, the model reduces to one having 130 dynamic degrees of freedom. airframe mass, both concentrated and distributed, is lumped at the grid points selected as the dynamic degrees of freedom. Structural damping of the airframe was not considered.

The primary vertical vibratory force coming from the rotor acts at grid point 55. The force has a magnitude of 1000 lb and a frequency of  $10.8~\rm{Hz}$  ('2/rev').

### NUMERICAL RESULTS AND DISCUSSIONS

Numerical results from the application of sensitivity analysis to a stick finite element model of the AH-1G helicopter airframe are presented and discussed here.

Finite Element Analysis Results:

A finite element analysis of the elastic line model was made using MSC/NASTRAN. The first few lowest natural frequencies obtained for the model are - 3.02 Hz (pylon pitch), 4.22 Hz (pylon roll), 6.80 Hz (1st airframe lateral bending), 7.85 Hz (1st airframe vertical bending), 16.70 Hz (2nd airframe lateral bending) and 17.10 Hz (2nd airframe vertical bending). The mode shapes corresponding to the vertical bending modes are shown in Figures (6 and 7). The first mode (frequency 7.85 Hz) has two nodes (zero displacement) on the airframe - one near the pilot seat and another near the middle of the tailboom. The second vertical bending mode (frequency 17.1 Hz) has three nodes - near grid points 6, 14, and 28.

The steady-state response of the airframe due to vertical excitation at a frequency of 10.8 Hz is shown in Figure (8). The response shape has two nodes (points of zero displacement) - one near grid point 2 and another near grid point 22. All other points on the airframe vibrate at various levels of acceleration depending on the amount of displacement of the airframe from the undeformed position.

The element strain energies associated with the forced response were also calculated. The distribution of strain energy in the fuselage and tailboom elements is shown in Figures (9-10) and discussed in a later section.

Sensitivity Analysis Results:

Using the strain energy criterion, the structural members which are most likely to influence the natural frequencies and the response were identified. Elements in the rear part of the fuselage and most of the elements in the tailboom were identified as likely candidates. The cross-sectional properties of the elements identified were related to design variables. In particular, design variable 'b' of the beam element was related to the area and moment of inertia of the cross-section (which are linear and cubic functions of b). A small increment was given to b to compute a new value of the design variable.

Constraints on the steady-state dynamic response displacements were imposed at the gun turret and pilot seat grid point locations (4 and 8, respectively). Because only vertical responses were of interest, only the vertical displacements were constrained. Although constraints on lateral and torsional displacements would ultimately also be required in a realistic design analysis, they were not considered in this study. However, they can be easily included.

The sensitivity coefficients for the selected constraints were obtained from the MSC/NASTRAN DMAP program which was discussed earlier. The sensitivity coefficients are plotted in a bar chart format in Figures (11-14). The numerical value of a coefficient indicates the amount of change in constraint value due to a small (positive) change in the design variable (identified by the element number, which also denotes the design variable number). A positive/negative value of a sensitivity coefficient means that an increase in the design variable results in an increase/decrease in the constraint value. To physically interpret the results it is useful to refer to the sensitivity of displacements  $(\Im x/\Im b)$  rather than the sensitivity of constraints  $(\Im h/\Im b)$ . These sensitivities differ only by a constant.

The results shown in Figures 11-12 indicate that the sensitivity coefficients related to the tailboom elements have magnitudes which are large compared to the fuselage elements. Consider the sign of these coefficients. In the tailboom region the coefficients are negative, whereas they are positive in the fuselage This means that an increment in a design variable associated with the members in the tailboom decreases the displacement at the pilot seat (and vice-versa) whereas an increment in a design variable in a fuselage member increases the displacement at the pilot seat (and This shows that the tailboom must be stiffened and/or the vice-versa). fuselage must be softened to reduce the dynamic response displacement at the pilot seat. The sensitivity coefficients obtained for constraints at the gun turret location are shown in Figures 13 and 14. The tailboom elements have coefficients which are an order of magnitude higher than those for the fuselage elements. This indicates that the tailboom elements should be significantly stiffened. The coefficients are negative for the fuselage and all elements in the tailboom (except for element number 1213 which has a positive coefficient). suggests that the elements of the tailboom and the fuselage (except 1213) require stiffening to reduce the dynamic response at the gun turret location. However, element 1213 requires a reduction in Hence, to satisfy the vibration constraint at the gun turret location a stiffening of the airframe structure is required, with an element with reduced stiffness at the junction of the fuselage and the rotor mast (grid 12) of the airframe. In summary, the tailboom requires a significant increase in stiffness to reduce the dynamic response at both the pilot seat and gun turret locations. Thus, rather straightforward considerations have provided the information about the portion of the airframe to be modified, order of magnitude of modification required, and the direction in which the modification (stiffen or soften) is required.

As the forced response of the airframe is a function of the natural frequencies and mode shapes of the structure as well as the excitation forces, any modification to the design variables to control the response will also bring about changes in the natural frequencies. also required. Constraints on the two lowest vertical bending modes (natural frequencies 7.85 and 17.1 Hz) of the airframe were considered here. Upper and lower limits on the first mode were specified at 7.0 and 8.5 Hz, respectively, and at 12.0 and 18.0 Hz, respectively, for the second mode. MSC/NASTRAN Rigid Formats 63 and 53 were used to obtain the sensitivity coefficients for the natural frequency constraints. The results are discussed in the following paragraph.

The sensitivity coefficients for the constraints imposed on the natural frequencies are plotted in Figures 15 and 16. The coefficients obtained all have positive values. The figures indicate that the coefficients related to the tailboom elements are large compared to the coefficients for most of the fuselage elements in the case of the first vertical bending mode. This shows that tailboom design strongly influences the natural frequency of the first vertical bending mode. In the case of the second vertical bending mode, some (aft) fuselage elements and (rear) tailboom elements have sensitivity coefficients larger than other elements of the airframe, and therefore they have a strong influence on the frequency of that mode. In both cases the coefficients are positive indicating that stiffening the elements increases the natural frequency, as might be expected.

### Interpretation of Results:

The calculation of sensitivity coefficients for a set of condtraints often constitutes a major computational effort in an optimization study. The sensitivity analysis results together with the dynamic characteristics of the airframe must be interpreted carefully to guide iterations to a low vibration design. Proper interpretation of the results will provide insight into the nature of the modifications required for the airframe and the feasibility of such modifications. The results presented above are interpreted and discussed below.

The steady-state response of the airframe is mainly due to excitation of the two lowest vertical bending modes (7.850 and 17.1 Hz) by the vertical force (10.8 Hz). The response shape resembles the first vertical bending mode, with the tailboom responding significantly more than the fuselage. The large motion of the tailboom may be attributed to the fact the tailboom is relatively soft compared to the fuselage. Therefore, to shift the natural frequencies and thereby change the response, the stiffness of the tailboom should be suitably changed. The sensitivity results also suggest that changes should be made to the tailboom design, that is, to increase the tailboom stiffness. Thus, the results on dynamic characteristics and sensitivity analysis are complementary.

Consideration of strain energy results together with sensitivity results can also be meaningful. In particular, compare the distribution of element strain energy densities in the forced response mode shape with the distribution of sensitivity coefficients in the The element strain energy densities in the tailboom are higher than those in the fuselage elements. This comparison indicates that elements with higher strain energies have higher magnitudes of sensitivity coefficients. Therefore, it would be beneficial to use both strain energy information and sensitivity results in the optimization procedure. There could be two possibilites here - one is to use the strain energy results to select design variables; another is to use the strain energy result to modify the design instead of using a more costly design sensitivity analysis. The later possibility is yet to be investigated. In this regard an explicit relation between the strain energy of elements and sensitivity coefficients would be useful.

The overall dynamics of the airframe has some bearing on the optimization of an airframe for vibration reduction. In a conservative dynamic system, the work done by external forces on a flexible structure is transformed into strain energy and kinetic energy. In a nonuniform structure, the distribution of these energies depends on the stiffness and mass distributions. Often a portion of a structure (for example, the tailboom of the AH-1G helicopter) may vibrate significantly more than other portions. In a sense the portion of the structure which vibrates most acts like a vibration absorber. Therefore, if one tries to reduce vibration in a certain portion of the airframe, some other portion of the airframe will vibrate excessively.

From the above discussion, the following possibilities offer themselves for reducing vibrations in the fuselage:

- 1. Soften the tailboom so that it acts like a vibration absorber.
- 2. Stiffen the tailboom and soften the fuselage to reduce vibration at the pilot seat.
- 3. Stiffen the tailboom and the fuselage and provide a soft spring-like interface structure between them to reduce vibration at the gun turret.

Clearly, these possibilities are not realistic in practice. However, they do suggest the types of modifications required for the airframe to satisfy the design constraints. The magnitudes of the modifications required can be obtained by interfacing the sensitivity analysis program with an optimizer. Careful selection of limits on design variables and constraints is needed, otherwise an optimizer may drive the design to an unrealistic configuration. Also, other types of constraints that must be imposed in a realistic airframe design should be included in the study. Therefore, the airframe optimization problem must be viewed in a broader perspective by considering the total helicopter system and not just a part of it.

### CONCLUSIONS

An initial study on design sensitivity analysis of rotorcraft airframe structures for vibration reduction has been made. A mathematical formulation for sensitivity analysis for constraints on steady-state forced response displacements was presented. The equations for the sensitivity coefficients were implemented as a new solution sequence in MSC/NASTRAN. Calculation of sensitivity coefficients was made using an elastic line model of the AH-1G helicopter airframe. The results of this preliminary study indicated the following:

- 1. Sensitivity coefficient results indicate that tailboom elements significantly influence the vibration response at the pilot seat and gun turret locations.
- 2. Sesitive elements of the airframe have higher element strain energies.
- 3. The first two vertical bending modes of the AH-1G airframe have a significant influence on the vertical response of the airframe under '2/rev' vertical rotor excitation loads.
- 4. Interpretation of the airframe dynamic characteristics together with the sensitivity analysis results has brought out the essential nature of modifications required in the AH-1G airframe to reduce vibration.

## DIRECTIONS FOR FUTURE WORK

The initial study on airframe sensitivity analysis indicates that there are several important aspects that must be considered. Based on the study, the following areas are identified for further investigation:

- 1. Consider constraints on static strength, forced response and natural frequencies simultaneously.
- 2. Interface an optimizer with the design sensitivity analysis
- 3. Study built-up finite element models.
- 4. Include airframe structural damping.
- 5. Include the effect of change of excitation force due to change in airframe flexibility.
- 6. Address problem of disjoint design space in forced response constraint formulation.
- 7. Consider a broader range of constraints (center-of-gravity movement of airframe, crash-loads, etc.,) for more effective use of optimization in actual helicopter design.

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## PERTINENT EQUATIONS

EQUATIONS FOR STEADY-STATE RESPONSE

$$MX + CX + KX = F$$
 $i\omega t$ 
 $i\omega t$ 
 $i\omega t$ 
 $i\omega t$ 

EQUATIONS FOR NATURAL MODES

$$MX + KX = 0$$

Where X = x e

UNDAMPED FORCED RESPONSE ELEMENT STRAIN ENERGY

$$U = \frac{1}{2} x k_e x$$

MODAL ELEMENT STRAIN ENERGY

$$U = \frac{1}{2} x k_e x$$

Figure 1.

## EQUATIONS FOR SENSITIVITY COEFFICIENTS

CONSTRAINTS  $\phi$  ON STEADY-STATE DYNAMIC DISPLACEMENTS:

$$\phi \equiv \frac{|X|}{|X_a|} - 1 \leqslant 0$$

STATE EQUATION FOR DYNAMIC DISPLACEMENTS:

$$h(b,x) = (-\omega^2 M + i\omega C + K) x - f = 0$$

Linear approximation to change in h due to change in b:

$$\partial h = \frac{\partial h}{\partial b} \partial b + \frac{\partial h}{\partial x} \partial x ALSO, \quad \partial x = \frac{\partial x}{\partial b} \partial b$$

EQUATIONS FOR SENSITIVITY COEFFICIENTS:

Figure 2.

## SENSITIVITY ANALYSIS FOR DYNAMIC RESPONSE USING MSC/NASTRAN DMAP SOLUTION SEQUENCE

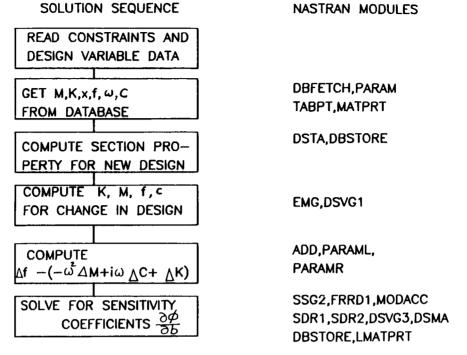


Figure 3.

### AIRFRAME STRUCTURE OF THE AH-1G HELICOPTER

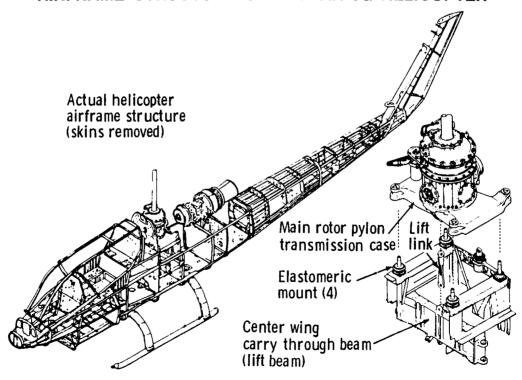


Figure 4.

## ELASTIC LINE (STICK) MODEL OF THE AH-1G AIRFRAME

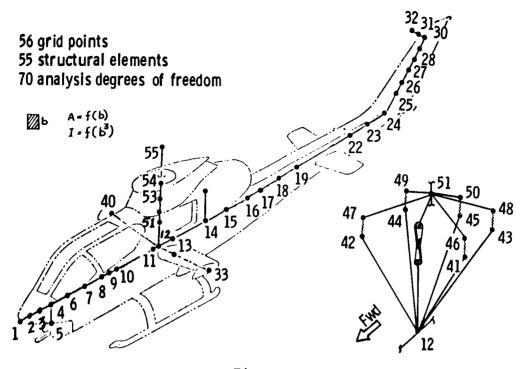


Figure 5.

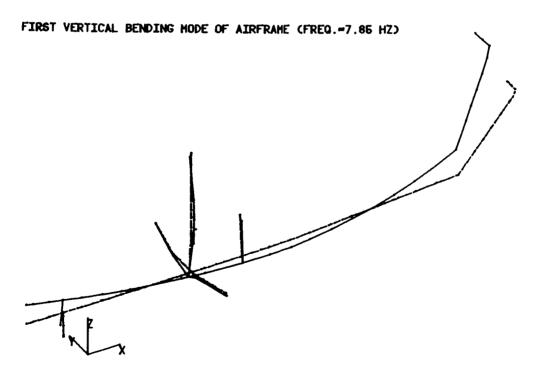


Figure 6.

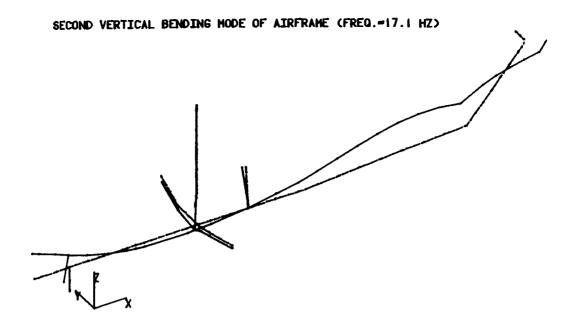


Figure 7.

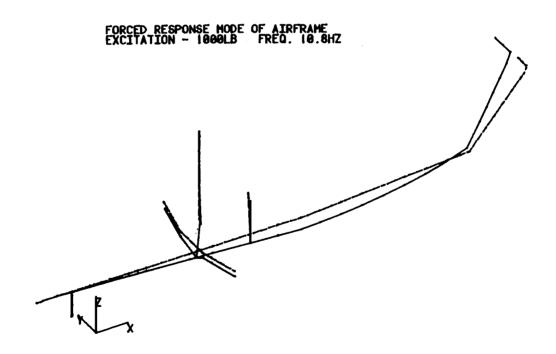


Figure 8.

## ELEMENT STRAIN ENERGY DENSITIES IN FUSELAGE FOR FORCED RESPONSE

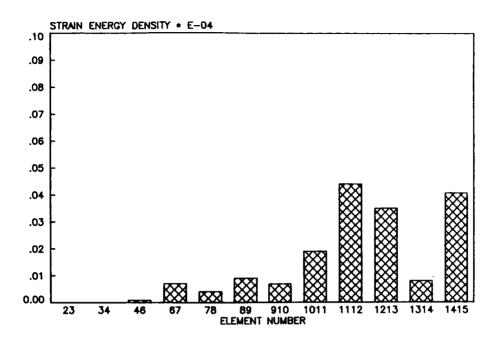


Figure 9.

## ELEMENT STRAIN ENERGY DENSITIES IN TAILBOOM FOR FORCED RESPONSE

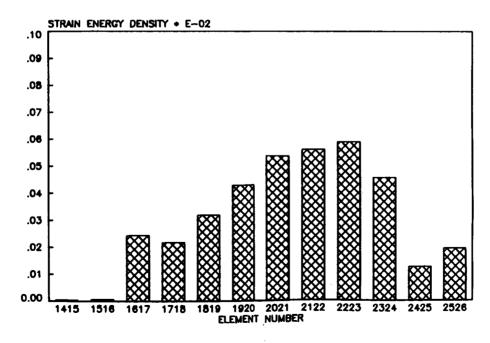


Figure 10.

# SENSITIVITY OF DYNAMIC DISPLACEMENT FOR PILOT SEAT LOC. W.R.T. FUSELAGE ELEMENTS COEFFICIENT COEFFICIENT

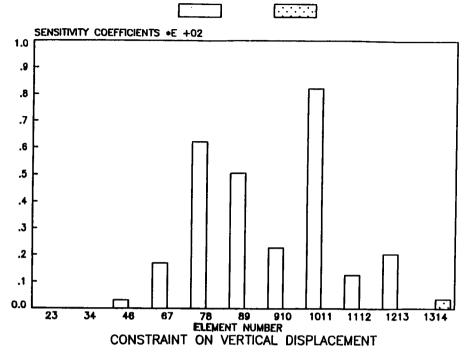


Figure 11.

# SENSITIVITY OF DYNAMIC DISPLACEMENT FOR PILOT SEAT LOC. W.R.T TAILBOOM ELEMENTS COEFFICIENT COEFFICIENT

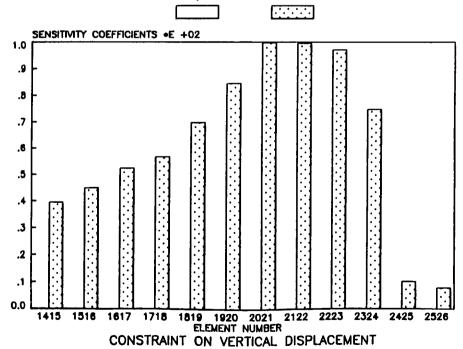


Figure 12.

# SENSITIVITY OF DYNAMIC DISPLACEMENT FOR GUN LOCATION W.R.T. FUSELAGE ELEMENTS COEFFICIENTS

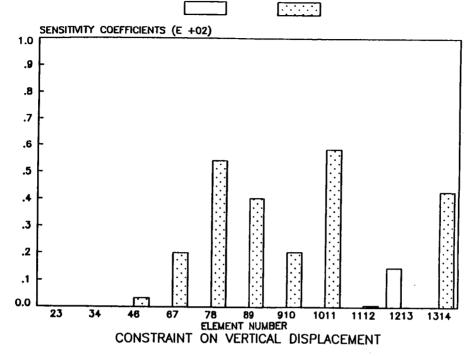


Figure 13.

## SENSITIVITY OF DYNAMIC DISPLACEMENT FOR GUN LOCATION W.R.T. TAILBOOM ELEMENTS COEFFICIENTS

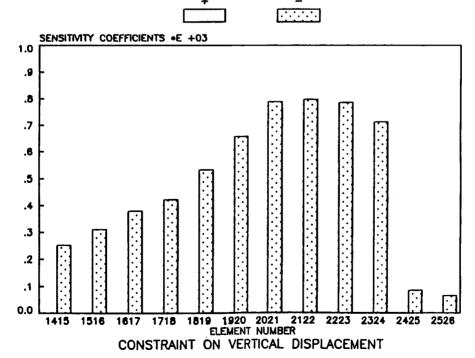


Figure 14.

## SENSITIVITY OF FREQUENCIES IN FIRST AND SECOND VERTICAL BENDING MODES MODE 1 MODE 2

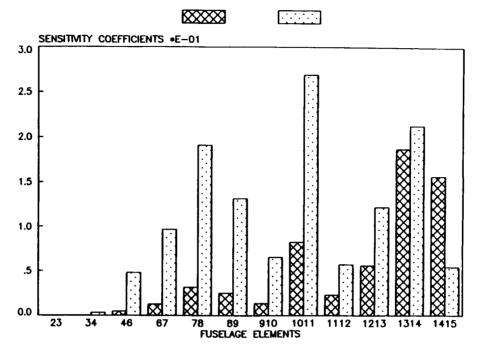


Figure 15.

## SENSITIVITY OF FREQUENCIES IN FIRST AND SECOND VERTICAL BENDING MODES MODE 1 MODE 2

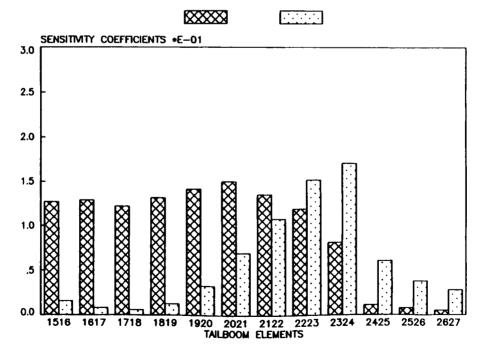


Figure 16.